HOSPITAL COSTS AND INFORMATION THEORY CASE MIX INDEXES: RESULTS FOR QUEENSLAND*

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The use of information theory as a basis for the construction of scalar case mix indexes for hospitals is well established but to date no results arising from an application of these indexes to Australian hospitals have been published. This paper provides a simplified explanation of the information theory approach and constructs the indexes for Queensland public hospitals. The usefulness of the indexes is then demonstrated with two applications. First, they are used to explain the variation in average cost per case between the hospitals in the study and are found to account for a small but statistically significant amount of such variation. Second, they are employed to provide estimates of state mean average and marginal costs by case type in Queensland. The resulting estimates are all both positive and plausible, characteristics not commonly found in estimates obtained using other techniques.

Keywords: information theory, hospital costs, case mix, index numbers, public hospitals

INTRODUCTION

Empirical analysis of cost behaviour in any industry must confront the problem of defining, measuring and classifying the industry's output. While this problem can be a difficult one to resolve for any industry, it is particularly acute for service industries which, by their nature, often produce intangible outputs.

With regard to hospitals, it is useful to tackle this problem by considering initially four broad categories of output (see Figure 1): inpatient treatment (the treatment of patients who are admitted to stay in hospital while being treated); outpatient treatment (the treatment of patients without admittance); teaching (the provision of education and training); and research (systematic inquiry aimed at expanding the stock of knowledge in medicine). Of these, inpatient treatment constitutes the primary output of hospitals — it is this which differentiates hospitals

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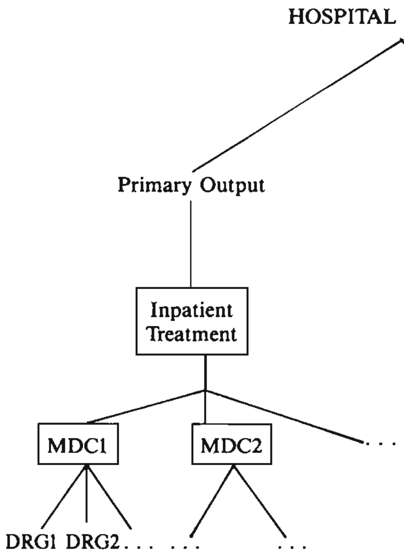
from other health care institutions.[1] The other three output categories are secondary in the sense that each can be produced in institutions other than hospitals.[2] Inpatient treatment is the focus of attention in this paper.

FIGURE 1



Within the broad category of inpatient treatment, further classification is necessary to obtain anything approaching internally homogeneous output categories. Inpatients differ in a range of dimensions, all of which can potentially affect the cost of their treatment. The nature of their illness is obviously an important dimension in this regard, as may be other factors such as age, sex and whether or not a surgical procedure is performed. The term 'case mix' has evolved to describe this phenomenon, referring to the mix of cases treated by a hospital classified according to those criteria which are significant in explaining the differences in resource usage between cases treated.[3]

Various criteria have been employed by different authors in classifying cases treated by hospitals, but more recently the classification of cases into what are known as Diagnosis-Related Groups (DRGs) has become prominent.[4] This scheme was developed by initially partitioning cases treated into predetermined Major Diagnostic Categories (MDCs) and then further subdividing each MDC to arrive at the DRGs (see Figure 1). The second version of this scheme began with 23 MDCs and contains a total of 467 DRGs. The DRG classification scheme has been employed by the United States (US) government for the last four years in its case mix sensitive hospital payment scheme for patients treated under the Medicare program for the aged.[5] It has also been used in Australia to classify the outputs of Victorian public hospitals.[6]

The necessity for a case mix classification scheme in analysing hospital costs and outputs is widely recognised and agreed upon, but there remain two divergent approaches to the incorporation of this information into hospital output measurement. One approach is to work at a disaggregated level with multiple output categories constructed on the basis of a case mix classification scheme such as DRGs. The second approach, although taking into account the diagnostic and perhaps other characteristics of patients, seeks to work at an aggregated level by constructing a single-valued measure of hospital case mix through the use of a case mix index. In contrast with the first approach, this approach does not result in multiple output categories but seeks to capture the influence of case mix in a scalar case mix index. Such an index collapses the multifarious output categories into a single, case mix sensitive index number for each hospital.

The purpose of this paper is to present some empirical results of an analysis of hospital costs using a particular type of index of hospital case mix, *viz.* the scalar indexes developed from information theory by Evans and Walker.[7] Accordingly, the next section of the paper is devoted to an overview of the construction of these indexes and an explanation of their meaning. The remainder of the paper is then concerned with empirical matters. After a discussion of the data employed, the empirically calculated information theory weights are presented. Following this, the usefulness of the information theory indexes is demonstrated in an analysis of hospital costs which provides estimates of average and marginal costs by case type.

INFORMATION THEORY CASE MIX INDEXES

Components of a Scalar Case Mix Index

A scalar case mix index attempts to provide a single-valued measure of the output composition of a hospital. The multiproduct nature of the hospital is taken into account in constructing the index by the use of weights incorporated into the aggregator function, such weights reflecting the heterogeneity between case mix categories. Hornbrook[8] identifies three components of such an index: a diagnostic classification scheme; a weighting scheme; and an aggregation formula.

Addressing the last of these first, aggregation formulae can generally be either linear or non-linear. "In practice, most authors assume a linear relationship. This assumption reflects the simplicity of a linear index and the lack of a priori or empirical support for a more complex formulation".[9]

A weighting scheme is necessary to establish relativities between the various case types and so allow meaningful aggregation. An important consideration in establishing these relativities is the objective to be achieved in constructing the index. If it is used in an analysis of hospital costs then the weights would presumably reflect the relative costliness of treating the various case types. If, however, the index is to be used as an indicator of the social benefit resulting from the treatment then the weights would reflect the relative social value placed on the treatment of each case type.

The third component of a scalar case mix index — a diagnostic classification scheme or, more generally, a case mix classification scheme — is necessary in order to establish the output categories over which aggregation is to be carried out. Note that the use of a single-valued index does not eliminate the need for such a classification scheme. On the contrary, a case mix classification scheme is an essential input into the construction of such an index.

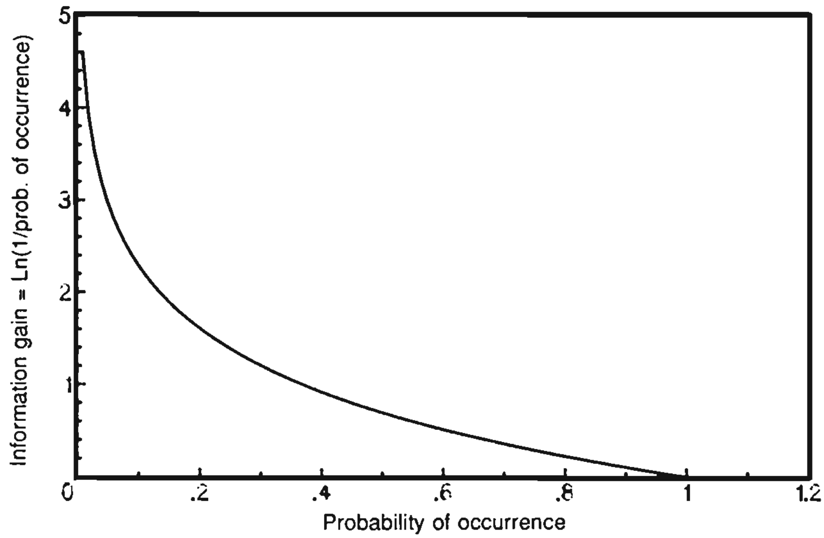
The Information Theory Indexes

Perhaps the most well known scalar case mix index is that developed by Evans and Walker based upon information theory. Employing the earlier work of Theil,[10] they postulated that the information gain from learning that an event has taken place is inversely related to the prior probability of that event occurring. If an event is almost certain to take place, i.e. it has a high probability of occurrence, then the information gain from learning that it has in fact taken place is relatively low, and vice versa. Quantitatively this information gain is measured as the log of the inverse of the probability of occurrence. Figure 2(a) illustrates the information gain as a function of the probability of occurrence. When two or more events are being considered, the information gain across all events is obtained as the probability-weighted sum of the individual gains.

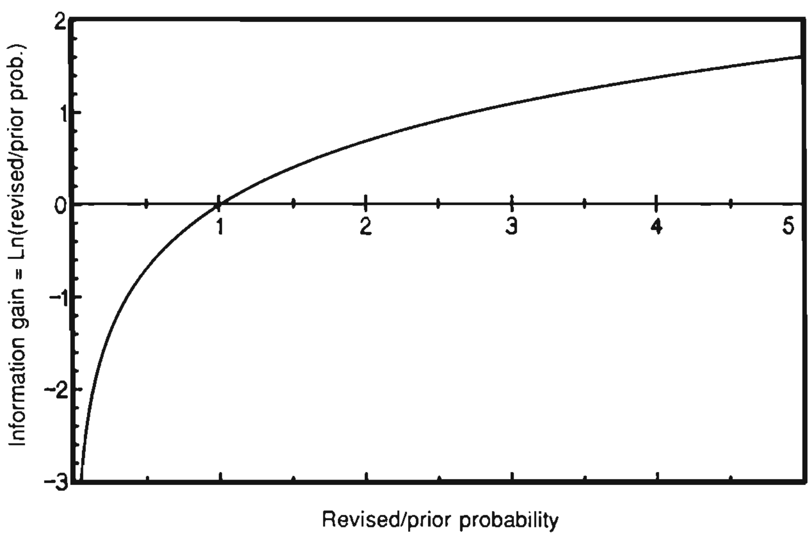
These concepts can also be used as a basis for measuring the information gain from learning that the probability of occurrence of an event has changed from the original probability. The information gain from learning of the revised probability of occurrence of an event is calculated as the difference in the logs of the revised and original probabilities, which is equal to the log of the ratio of the revised and original probabilities. Figure 2(b) illustrates the information gain from learning of the revised probability of an event expressed as a function of the ratio of revised and original probabilities. Note that if the revised

FIGURE 2

(a) Information gain as a function of probability of occurrence of an event



(b) Information gain from learning of revised probability of occurrence of an event as a function of ratio of revised to prior probability of occurrence.



probability is less than the original probability, this ratio is less than unity and the information gain is negative, i.e. since the probability of occurrence has fallen we now have less information. Again, where two or more events are being considered, the information gain from learning of the revised probabilities of occurrence of all the events is the probability-weighted sum of the individual gains where the weights are the revised probabilities.

Evans and Walker utilised the foregoing concepts in devising a weighting scheme for use in a scalar index of hospital case mix. In this context, the 'prior probability of an event' refers to the probability that any individual hospital in the system will treat the next case admitted to the system over any given time period.

Evans and Walker construct two information theory indexes based on two different values for this prior probability. The first assumes no prior knowledge of the distribution of cases among hospitals and so takes (1/N) as the prior probability of a case being admitted to any hospital where N is the number of active treatment hospitals, i.e. the probability of a case going to any hospital is the same for all hospitals and equal to the universe of the number of hospitals.

The second index differs from the first in that prior knowledge of the proportion of cases actually treated by any particular hospital is incorporated, i.e. the prior probability is now the actual proportion of all cases in the system treated by a hospital. This is obviously sensitive to the volume of cases treated by a hospital — the larger the proportion of any given volume of cases treated by a hospital, the larger is the probability that it is going to treat any particular case admitted to the hospital system.

The weights for the scalar case mix index are then calculated as probability-weighted sums of information gains from learning of the actual distribution of cases between hospitals disaggregated by case type. For each of the two indexes the revised probability of an event is the proportion of all cases *of a particular type*, e.g. malignant neoplasms, in the system being treated by a particular hospital. For the first index, then, the information gain is obtained as the ratio of this probability to the prior probability (1/N). For the second index, the information gain is the ratio of this probability to the actual proportion of all cases in the system treated by the hospital.[11] For each index, the weight for a particular case type is calculated as the probability-weighted sum across hospitals of the information gain for each hospital where the weights are the revised probabilities.

In this way a weight is calculated for each case mix category so there are as many weights as there are case mix categories. The weights are then standardised to have a mean of unity.[12] Finally, the scalar case mix indexes for each hospital are obtained by using these weights in a linear aggregation of each hospital's case mix proportions, i.e. the indexes for a hospital are calculated as a weighted linear sum of the proportions of that hospital's cases which are treated in each of the case mix categories.

In interpreting the index number for a hospital it is important to understand the interpretation of the weights used in its construction. Recall that if the revised probabilities equal the prior probabilities in either type of index the information gain is zero (see Figure 2). Now if this were true for any case mix category then the weight for that category would be zero. In general, for both indexes, the more closely the actual distribution of cases within a particular case mix category matches the assumed prior distribution, the lower is the information gain (and hence the weight). Conversely, the more concentrated the cases in a smaller number of hospitals, the larger is the information gain (and hence the weight). The following crucial hypothesis then establishes a nexus between concentration, these measures of information gain and case complexity: "If concentration is associated with complexity, then the expected information gain of a specific case type is a measure of its complexity".[13] A larger information gain, and hence a higher weight, indicates a more *complex* case type.

While these case mix indexes represent an ingenious application of information theory to the problem of measuring hospital output, they suffer from a number of limitations which need to be borne in mind. First, as with all single-valued indexes, identical values of the index can be obtained for hospitals with different underlying case mixes.[14] Second, the hypothesis that a higher concentration of cases in a smaller number of hospitals implies higher complexity may confuse complexity with rarity.[15] The fact that most cases of a particular type are treated in a small number of hospitals may indicate that the condition is rare rather than complex. This problem is more serious the more disaggregated the case mix classification scheme which is employed, for when there are large numbers of case mix categories some case types will almost certainly be rare but not complex. Tatchell[16] produced some empirical evidence on this matter by constructing the weights for New Zealand public hospitals using 50 and 150 case mix categories based on the International Classification of Diseases (ICD) codes.

> The complexity values derived from the larger list confirmed . . . that there was some 'loss of reliability' in the information measure with larger number [sic] of diagnoses because of the increased chance of encountering rare diseases. Values for infective and parasitic diseases, for example (of which there are 44 in the longer 150 item list), were considerably higher than expected, the result of their rarity rather than their complexity.[17]

A third problem is that the information theory case mix indexes do not capture variations in the complexity of cases *within* any particular case mix category. This is actually a problem with case mix classification schemes rather than with the technique of constructing the index and is a reflection of the general difficulty of devising a set of internally homogeneous output categories for hospitals. Within-group heterogeneity has also been raised as criticism of DRGs. In this context it should be noted that scalar case mix indexes in general will have built into them any weaknesses or limitations of the underlying output categories used in their construction.

Finally, the indexes assume what Klastorin and Watts have called "functional homogeneity"[18] — that the relationship embodied in the index is sufficiently similar across institutions to justify using the same aggregation formula and the same weights. Functional homogeneity exists if both the functional form of the index (linear or non-linear) and the weights for the case mix categories can be used for all hospitals in the sample. But if various sub-groups of hospitals within the sample, e.g. teaching and non-teaching hospitals or metropolitan and country hospitals, differ systematically for some reason which renders the use of the same functional form and/or weights inappropriate then separate indexes should be constructed for each sub-group.

Having described the information theory indexes and considered various problems with them, attention is now directed to empirical application. This paper contains the first published results arising from an application of the information theory indexes to Australian data. Following a brief description of the data, these empirical results are presented.

THE DATA

The sample of hospitals used in the present study consists of 121 Queensland public hospitals with data for the financial year 1977-78. The sample includes all public hospitals which treated inpatients in the year and for which reliable data were available.[19]

The data were drawn from two separate statistical collections held by the Queensland Department of Health — the Hospital Morbidity Data and the Hospital Finance Data. The Hospital Morbidity Data comprise a unit record for each *discharge* (live or dead) from every acute hospital in Queensland. The unit record on each episode of hospitalisation contains information on, among other things, date of admission and discharge, demographic information, hospital identity, and summary information on principal diagnosis and principal medical procedure (if any). The Hospital Finance Data comprise aggregated budgetary information about each public hospital. The cost data available from this collection relate to maintenance costs and 'interest and redemption' but the latter bear no necessary relationship to the economic cost of capital.

In constructing the information theory case mix indexes, two case mix classification schemes have been adopted. One of these is an 18 diagnostic category specification using the 17 major chapter headings plus the supplementary classifications of the eighth revision of the ICD (ICD-8). The other is a more disaggregated 47 diagnostic category classification used in constructing a relative stay index for Queensland hospitals also based on ICD categories.[20] The mean proportion of cases treated in each category together with the coefficient of variation[21] and the maximum value are reported for the 18 and 47 diagnostic category classifications in Tables 1 and 2 respectively.

TABLE 1

MEAN, COEFFICIENT OF VARIATION AND MAXIMUM VALUE OF CASE MIX PROPORTIONS, 18 DIAGNOSTIC CATEGORIES, QUEENSLAND PUBLIC HOSPITALS, 1977-78*

No	Diagnostic Category	Mean (%)	C.V.	Max (%)
1	Infectious & Parasitic Diseases (000-136)	4.96	.6289	18.2
2	Neoplasms (140-239)	2.04	1.1022	13.9
3	Endocrine, Nutritional & Metabolic (240-279)	1.59	.5958	6.0
4	Blood (280-289)	0.43	.9286	2.6
5	Mental Disorders (290-315)	3.61	.6830	13.7
6	Nervous System (320-389)	3.88	.5303	14.5
7	Circulatory System (390-458)	7.82	.5232	25.6
8	Respiratory System (460-519)	14.45	.4546	44.2
9	Digestive System (520-577)	6.32	.5210	14.8
10	Genito-Urinary System (580-629)	5.32	.6576	21.1
11	Complications of Pregnancy, Childbirth & Puerperium (630-678)	12.18	1.1484	96.1
12	Skin & Subcutaneous Tissue (680-709)	2.72	.6459	9.5
13	Musculoskeletal System (710-738)	2.55	.5987	8.3
14	Congenital Anomalies (740-759)	0.42	2.4833	8.4
15	Causes of Perinatal Morbidity and Mortality (760-779)	0.36	2.1406	5.5
16	Symptoms & Ill-defined (780-796)	12.94	.5334	42.9
17	Accidents, Poisonings & Violence (N800-N999)	15.47	.3538	32.1
18	Supplementary Classifications (Y00-Y89)	2.95	.7284	11.1

Note: * ICD-8 codes in parentheses
Source: Hospital Morbidity Data, Queensland Department of Health.

Some descriptive statistics on cost and volume variables for hospitals in the sample are presented in Table 3.[22] The cost data pertain to maintenance costs only, i.e. they exclude interest and redemption, and have been adjusted to exclude estimates of outpatient costs. The ranges of values of these variables indicate the diversity of hospitals in the sample with respect to these measures. For example, cost per case ranges from just under $35 up to in excess of $1,300, and the case flow rate ranges from 1.5 to nearly 78.

TABLE 2

MEAN, COEFFICIENT OF VARIATION AND MAXIMUM VALUE OF CASE MIX PROPORTIONS, 47 DIAGNOSTIC CATEGORIES, QUEENSLAND PUBLIC HOSPITALS, 1977-78*

No	Diagnostic Category	Mean (%)	C.V.	Max (%)
1	Invest. Procedures, Healthy (Y00-Y89)	2.95	.7284	11.1
2	Infectious & Parasitic (000-007, 010-136)	2.14	.7412	10.6
3	Enteritis, Diarrhoeal Disease (008-009)	2.82	.8101	12.3
4	Malignant Neoplasms (140-209)	1.58	1.1966	12.6
5	Benign Neoplasms (210-239)	0.46	1.0880	2.9
6	Endocrine & Metabolic (240-279)	1.59	.5958	6.0
7	Blood (280-289)	0.43	.9286	2.6
8	Psychiatric (290-315)	3.61	.6830	13.7
9	Other CNS & Nerves (320-358)	2.19	.5871	9.1
10	Eye & Ear (360-389)	1.69	.9735	9.6
11	Other Heart, Hypertension (390-404, 411-426, 428-429)	2.84	.8108	15.0
12	Acute Myocardial Infarction (410)	0.87	.8286	2.9
13	Symptomatic Heart Disease (427)	1.70	.8883	12.5
14	Cerebrovascular Disease (430-438)	1.06	.7524	3.6
15	Circulation (440-458)	1.36	.7098	4.9
16	Upper Respiratory (460-474)	4.49	.7524	16.5
17	Pneumonia (480-486)	2.37	1.2382	22.6
18	Bronchitis, Emphysema, Asthma (490-493)	4.49	.6625	17.0
19	Tonsils & Adenoids (500)	0.88	1.6182	8.3
20	Other Respiratory (501-519)	2.24	1.0263	12.2
21	Dental (520-529)	0.55	1.3040	3.6
22	Upper Gastrointestinal (530-537)	1.39	.8117	6.4
23	Appendicitis (540-543)	1.16	.9544	6.6
24	Hernia (550-553)	0.89	.9962	5.3
25	Other Gastrointestinal (560-577)	2.33	.5902	5.5
26	Nephritis & Nephrosis (580-584)	0.55	3.6483	15.4
27	Other Urinary (590-599)	1.62	.5627	4.2
28	Male Genital (600-607)	0.61	.7998	2.8
29	Other Female Genital (610-625, 627-629)	1.97	.9163	10.6
30	Disorders of Menstruation (626)	0.58	.9911	2.3
31	Complications of Pregnancy & Puerperium (630-639, 670-678)	3.48	.8781	25.9
32	Abortion (640-645)	0.80	.7714	4.2
33	Normal Delivery (650)	7.24	1.3815	79.2
34	Delivery Complications (651-662)	0.65	3.6028	25.5
35	Skin Disease (680-709)	2.72	.6459	9.5
36	Orthopaedic (710-738)	2.55	.5987	8.3

TABLE 2 (Continued)

No	Diagnostic Category	Mean (%)	C.V.	Max (%)
37	Congenital Malformation (740-759)	0.42	2.4833	8.4
38	Perinatal (760-776, 778-779)	0.22	2.2550	3.8
39	Immaturity (777)	0.14	2.8303	3.7
40	Symptoms, Ill-defined (780-793, 795)	11.72	.5636	42.9
41	Long Stay, Ill-defined (794, 796)	1.21	1.1639	11.5
42	Other Fractures (Excl. Femur) (N800-N819, N821-N829)	3.78	.6863	21.4
43	Fracture of Neck of Femur (N820)	0.20	1.5147	2.1
44	Dislocations (N830-N848)	0.83	.7967	3.4
45	Internal Injury (N850-N869)	3.04	.5985	8.8
46	External Injury (N870-N959)	4.54	.5849	18.2
47	Poisoning (N960-N999)	3.07	.5528	10.5

Note: *ICD-8 codes in parentheses
Source: Hospital Morbidity Data, Queensland Department of Health.

TABLE 3

MEAN, STANDARD DEVIATION, COEFFICIENT OF VARIATION AND RANGE OF COST AND VOLUME VARIABLES, QUEENSLAND PUBLIC HOSPITALS, 1977-78

	Mean	C.V.	Range Min	Range Max
Average Cost per Case ($)	546.83	.44	34.71	1,361.16
Average Cost per Day ($)	79.93	.45	16.71	220.09
Av. Length of Stay (days)	7.44	.48	1.00	19.48
Occupancy*	.43	.47	.006	1.075**
Case Flow Rate†	23.33	.61	1.50	77.70
Inpatients	2,725	2.08	14	39,907
Beds	98	1.79	2	1,234

Notes: * Occupancy is the proportion of available bed days which are occupied during the year.
** This figure relates to a maternity hospital where days of care to "qualified babies" are added to the occupied bed days provided to the mother.
† Case Flow Rate is the number of cases treated per bed.

Source: Hospital Finance Data, Queensland Department of Health.

EMPIRICAL RESULTS

Information Theory Case Mix Index Weights

The standardised complexity values (the weights) for each of the two types of information theory index were derived for both the 18 and 47 diagnostic classifications.[23] The use of two diagnostic classifications provides a check on the consistency of the complexity rankings produced by the information theory weights. It also provides some insight into whether rarity becomes a problem when using the more disaggregated diagnostic classification.

A comparison of the categories with the four lowest and the four highest standardised complexity values from each diagnostic classification scheme is provided in Table 4. Two main points emerge from this comparison. First, the Type 1 and Type 2 indexes vary to some degree in their complexity rankings for any given diagnostic classification scheme. For example, comparing the complexity values from the 47 diagnostic categories, the two types of index have no categories in common in the 'four lowest' list but produce the same categories in the 'four highest' list. In general, though, there is a statistically significant correlation between the rankings produced by the two types of index. The Spearman rank correlation coefficients are 0.65 and 0.44 for the 18 and 47 diagnostic category classifications respectively (both significant at the five per cent level).

TABLE 4

DIAGNOSTIC CATEGORIES WITH HIGHEST AND LOWEST COMPLEXITY VALUES, QUEENSLAND PUBLIC HOSPITALS, 1977-78*

Categories with four lowest complexity values — 18 DCs	
Type 1 Index	Type 2 Index
1 Infectious and Parasitic Diseases (0.52) 16 Symptoms and Ill-Defined (0.56) 8 Respiratory System (0.58) 12 Skin and Subcutaneous Tissue (0.68)	17 Accidents, Poisonings and Violence (0.29) 3 Endocrine, Nutritional and Metabolic (0.33) 9 Digestive System (0.35) 16 Symptoms and Ill-Defined (0.47)
Categories with four highest complexity values — 18DCs	
Type 1 Index	Type 2 Index
15 Causes of Perinatal Morbidity and Mortality (1.73) 14 Congenital Anomalies (1.70) 2 Neoplasms (1.62) 10 Genito-Urinary System (1.41)	15 Causes of Perinatal Morbidity and Mortality (4.15) 14 Congenital Anomalies (3.47) 11 Complications of Pregnancy, Childbirth and Puerperium (2.72) 5 Mental Disorders (1.61)

TABLE 4 (Continued)

Categories with four lowest complexity values — 47 DCs	
Type 1 Index	Type 2 Index
16 Upper Respiratory (0.40) 3 Enteritis, Diarrhoeal Disease (0.41) 17 Pneumonia (0.49) 40 Symptoms, Ill-Defined (0.53)	6 Endocrine and Metabolic (0.25) 47 Poisoning (0.27) 42 Other Fractures (Excl. Femur) (0.30) 9 Other CNS and Nerves (0.33)
Categories with four highest complexity values — 47 DCs	
Type 1 Index	Type 2 Index
26 Nephritis and Nephrosis (2.37) 39 Immaturity (1.92) 34 Delivery Complications (1.70) 38 Perinatal (1.63)	34 Delivery Complications (4.75) 39 Immaturity (4.47) 38 Perinatal (3.08) 26 Nephritis and Nephrosis (2.82)

Note: * Numbers in brackets are standardised complexity values.
Source: See Appendix.

The second main point which emerges from this comparison is that, for any given type of index, the results from the two diagnostic classifications are highly consistent. The categories with the lowest complexity values in the 47 diagnostic category classification are all subsets of the lowest complexity value categories in the 18 diagnostic category classification, and similarly for the high complexity value categories. This result is pleasing for it indicates that the 47 diagnostic category classification is not sufficiently disaggregated to be confusing rarity with complexity.

The complexity rankings produced by each type of index are intuitively plausible. They indicate the highly complex case types to be illnesses associated with renal disease, neoplasms, delivery complications, perinatal problems and immaturity. The results are also generally in accord with those produced by Evans and Walker who found that

> the highest complexities are recorded by list numbers (63) nephritis and nephrosis, (35) diseases of the eye, (2) poliomyelitis and encephalitis and (6) - (18) malignant neoplasms. At the bottom end are the variants of upper respiratory disease, skin infections, and stomach troubles.[24]

These authors were working with a 98 diagnostic category classification. Horn and Schumacher[25] also found that the information theory complexity measure correlated very highly with an independently constructed clinical measure of complexity.

There are, however, some differences between the rankings produced here and those obtained by Schapper[26] for a sample of Western Australian hospitals using 50 diagnostic categories. While it is not clear whether Schapper is working with Type 1 or Type 2 weights, only two of his 'four highest' categories (diseases of newborn and delivery with complications) appear in the 'four highest' in Table 4 using 47 diagnostic categories in either type of index. In the 'four lowest' rankings there are no common categories. The different results may be due to the fact that Schapper included only Perth metropolitan hospitals in his study and that deaths were included in a fifty-first category of their own.

Hospital Cost Analysis

The standardised complexity values provide the weights used in the construction of the information theory case mix index for hospitals. The value of the index for each hospital is obtained as the weighted sum of its case mix proportions. Since two different diagnostic classification schemes are employed and two types of index are constructed, four case complexity index numbers are produced for each hospital in this study. While the complete set of resulting index numbers is not reproduced here, Table 5 tabulates the minimum and maximum values of each index and the size of the hospital (measured in beds) which recorded the particular result. The same hospital scored the highest complexity value in three of the four indexes, while two hospitals each recorded the lowest complexity values of two of the indexes.

TABLE 5

MINIMUM AND MAXIMUM VALUES FOR INFORMATION THEORY CASE MIX INDEXES, QUEENSLAND PUBLIC HOSPITALS, 1977-78*

Index**	Minimum	Maximum
X181	0.67 (12)	1.14 (1,234)
X182	0.58 (4)	2.72 (80)
X471	0.63 (12)	1.24 (80)
X472	0.59 (4)	2.80 (80)

Notes: * Figures in brackets are sizes of hospitals (measured by number of beds) with the particular result.
** X181 = information theory index based on 18 diagnostic categories and first information theory index formula. The other symbols in this column have an analogous interpretation.

Source: Hospital Morbidity Data and information theory index calculations.

How useful, then, are these indexes? One important application relates to the explanation of variations in average cost per case between hospitals. Table 6 contains the correlation matrix for the four information theory indexes and average cost per case for Queensland public hospitals. Note first of all that the type of index employed (Type 1 or Type 2) has a much more important influence on the value of a hospital's complexity number than the diagnostic classification scheme. Indexes constructed using the same type of weights correlate much more closely than indexes constructed using different types of weights. The two Type 1 indexes have a correlation coefficient of 0.97 and the two Type 2 indexes have one of 0.98. There is, however, much less correlation between any of the Type 1 and Type 2 indexes. The correlation coefficient between the two types of index using 47 diagnostic categories is 0.67, and is 0.61 using 18 diagnostic categories.

TABLE 6

CORRELATION MATRIX, INFORMATION THEORY INDEXES AND AVERAGE COST PER CASE

	X181	X182	X471	X472	ACC
X181	1.00	.61	.97	.62	.38
X182		1.00	.64	.98	.09
X471			1.00	.67	.34
X472				1.00	.09
ACC					1.00

Source: Regression Results.

Turning to the correlations between each of the indexes and average cost per case, the type of index again has a more important influence than the diagnostic classification scheme. The Type 1 indexes have correlation coefficients with average cost per case of 0.38 and 0.34 using 18 and 47 diagnostic categories respectively, compared with 0.09 for each of the Type 2 indexes.

The performance of the two types of index in explaining inter-hospital variation in average cost per case is further investigated by undertaking an ordinary least squares bivariate regression of average cost per case on each of the case mix indexes. The results are presented in Table 7. The Type 1 indexes (X181 and X471) outperform the Type 2 indexes (X182 and X472), explaining 14 and 11 per cent of the variation in average cost per case respectively (after adjustment for degrees of freedom). The Type 2 indexes, however, actually have negative adjusted R^2 values and are statistically insignificant. All coefficients have the expected positive sign indicating that hospitals with a higher complexity index have a higher predicted average cost per case.

TABLE 7

PARAMETER ESTIMATES OBTAINED FROM BIVARIATE REGRESSION OF AVERAGE COST PER CASE AND INFORMATION THEORY CASE MIX INDEXES*

	X181	X471	X182	X472
Constant	−377.44	−99.60	484.85	484.34
Estimated Coefficient	1039.24	759.68	67.41	70.55
	(4.48**)	(3.97**)	(0.96)	(0.94)
Adj. R^2	0.14	0.11	−0.001	−0.001
SEE†	221.93	225.43	238.99	239.02

Notes: * t-values in parentheses; 119 degrees of freedom for each equation.
** significant at 1 per cent level.
† SEE = standard error of estimate.

Source: Regression results.

These results, of course, are not put forward as being estimated from a completely specified hospital cost function. Clearly there are other factors of importance which may explain differences in average cost per case but which are not included here, e.g. scale and utilisation. An analysis of these factors, however, is outside the scope of this paper.

The superior performance of the Type 1 index noted above was also found by Evans and Walker: "By far the strongest variable . . . is the first definition of complexity . . .".[27] Tatchell, who also constructed four indexes based on two different diagnostic classifications, found that "the two measures based on the assumption of no prior knowledge of the hospital system . . . appear to perform the better of the four measures . . . ".[28] The Type 1 index is that which assumes no prior knowledge. Horn and Schumacher reported that the Type 2 index "did not perform as well in the original regression equations"[29] and consequently excluded it from the remainder of their study. Watts and Klastorin[30] found their information theory index a poor explanator of average cost per case but it is not clear from their paper which index (Type 1 or Type 2) was constructed. Hardwick[31] included only the Type 1 index in her study, finding that this index on its own explained 24 per cent of the variation in average cost per case (adjusted R^2 = 0.24) for 111 acute care hospitals in Alberta, Canada, for the year 1978-79.

Attention is now directed to a second question concerning the usefulness of information theory indexes of hospital output, *viz.* whether they can be utilised to produce plausible estimates of state mean average and marginal costs by case type for the diagnostic categories used in their construction. Given the superiority of the Type 1 index in the foregoing results, these estimates will be obtained using only this type of index.

The first step in obtaining these estimates is to re-estimate the bivariate relationships between average cost per case and the two Type 1 case mix indexes (X181 and X471) with the constant term suppressed. This is equivalent to constraining the total cost function for each case type to be linear and to pass through the origin, and constrains average and marginal costs to be equal and constant. Each parameter estimate[32] is then multiplied by the relevant standardised complexity weights (see Appendix) to obtain estimates of average and marginal costs by case type.

TABLE 8

VALUES OF AVERAGE AND MARGINAL COST IMPLIED BY INFORMATION THEORY INDEX REGRESSION RESULTS, 18 DIAGNOSTIC CATEGORIES, QUEENSLAND PUBLIC HOSPITALS, 1977-78

No	Diagnostic Category	Implied Average and Marginal Cost per Case ($)
1	Infectious & Parasitic Diseases	323.98
2	Neoplasms	1,004.05
3	Endocrine, Nutritional & Metabolic	531.26
4	Blood	592.44
5	Mental Disorders	861.11
6	Nervous System	687.16
7	Circulatory System	638.28
8	Respiratory System	358.79
9	Digestive System	586.39
10	Genito-Urinary System	874.74
11	Complications of Pregnancy, Childbirth & Puerperium	684.19
12	Skin & Subcutaneous Tissue	423.31
13	Musculoskeletal System	687.96
14	Congenital Anomalies	1,053.58
15	Causes of Perinatal Morbidity & Mortality	1,067.62
16	Symptoms & Ill-defined	344.47
17	Accidents, Poisonings & Violence	484.07
18	Supplementary Classifications	667.83

Sources: Appendix and regression results.

TABLE 9

VALUES OF AVERAGE AND MARGINAL COST IMPLIED BY INFORMATION THEORY REGRESSION RESULTS, 47 DIAGNOSTIC CATEGORIES, QUEENSLAND PUBLIC HOSPITALS, 1977-78

No	Diagnostic Category	Implied Average and Marginal Cost per Case ($)
1	Investigations, Procedures, Healthy	652.68
2	Infectious & Parasitic	412.69
3	Enteritis, Diarrhoeal Disease	262.36
4	Malignant Neoplasms	1,043.02
5	Benign Neoplasms	785.45
6	Endocrine & Metabolic	519.21
7	Blood	579.01
8	Psychiatric	841.58
9	Other CNS & Nerves	598.85
10	Eye & Ear	802.11
11	Other Heart, Hypertension	587.48
12	Acute Myocardial Infarction	698.42
13	Symptomatic Heart Disease	531.71
14	Cerebrovascular Disease	725.16
15	Circulation	786.54
16	Upper Respiratory	257.53
17	Pneumonia	318.70
18	Bronchitis, Emphysema, Asthma	419.34
19	Tonsils & Adenoids	703.91
20	Other Respiratory	481.99
21	Dental	579.70
22	Upper Gastrointestinal	613.25
23	Appendicitis	542.93
24	Hernia	644.86
25	Other Gastrointestinal	643.01
26	Nephritis & Nephrosis	1,524.57
27	Other Urinary	602.55
28	Male Genital	712.03
29	Other Female Genital	653.34
30	Disorders of Menstruation	766.84
31	Complications, Pregnancy & Puerperium	574.06
32	Abortion	558.99
33	Normal Delivery	752.21
34	Delivery Complications	1,093.17
35	Skin Disease	413.71
36	Orthopaedic	672.36

TABLE 9 (Continued)

No	Diagnostic Category	Implied Average and Marginal Cost per Case ($)
37	Congenital Malformation	1,029.69
38	Perinatal	1,049.85
39	Immaturity	1,236.29
40	Symptoms, Ill-defined	340.49
41	Long Stay, Ill-defined	399.48
42	Other Fractures (Excl. Femur)	589.67
43	Fracture of Neck of Femur	939.29
44	Dislocations	433.17
45	Internal Injury	475.32
46	External Injury	390.69
47	Poisoning	457.67

Source: Appendix and regression results.

The average (equals marginal) cost estimates for the 18 and 47 diagnostic categories are presented in Tables 8 and 9 respectively.[33] It is immediately evident that this approach produces positive, plausible average (equals marginal) cost estimates by case type. The 18 diagnostic category estimates range from $323.98 (category 1) up to $1,067.62 (category 18) while the 47 diagnostic category estimates range from $257.53 (category 16) up to $1,524.57 (category 26). Given the state mean cost per case of $546.83 for 1977-78, these figures are generally quite reasonable. Because of the dependence on the relative values of the underlying weights, the credibility of any particular estimate depends directly on the credibility of the underlying case complexity weight. Such weights have already been found to be generally tenable.

The significance of obtaining positive estimates of average and marginal costs by case type using the information theory indexes warrants emphasis. Empirical estimates of hospital cost functions have often been plagued by negative values for at least some of the case type cost estimates, even when sophisticated statistical techniques such as principal components analysis have been used.[34] The information theory indexes can then serve a useful purpose in providing positive estimates of such costs.

CONCLUSION

This paper has provided a simplified explanation of the information theory case mix indexes of hospital output and has empirically constructed such indexes using data on Queensland public hospitals. These are the first published results arising from the application of such indexes to Australian data.

To demonstrate the usefulness of these indexes, they were then employed to explain the variation in average cost per case between Queensland public hospitals and to construct estimates of state mean average and marginal costs by case type. The case mix indexes constructed on the assumption of no prior knowledge of the distribution of cases between hospitals have been found to explain a small (11 to 14 per cent) but statistically significant amount of inter-hospital variation in average cost per case. They also give rise to positive, credible estimates of average and marginal costs by case type.

It is recognised that the results presented here arise from a limited exercise in that a fully specified hospital cost function has not been estimated. The estimation of a fully specified such function is, of course, an important matter since it will also incorporate the effects of other factors such as scale and utilisation. This task, however, lies outside the scope of the present paper.

APPENDIX

STANDARDISED COMPLEXITY WEIGHTS, QUEENSLAND PUBLIC HOSPITALS, 1977-78

18 Diagnostic Categories

No	Diagnostic Category	Type 1	Type 2
1	Infectious & Parasitic Diseases	0.52*	0.92
2	Neoplasms	1.62**	1.22
3	Endocrine, Nutritional & Metabolic	0.86	0.33*
4	Blood	0.96	0.62
5	Mental Disorders	1.39	1.61**
6	Nervous System	1.11	0.66
7	Circulatory System	1.03	0.71
8	Respiratory System	0.58*	0.85
9	Digestive System	0.95	0.35*
10	Genito-Urinary System	1.41**	0.87
11	Complications of Pregnancy, Childbirth & Puerperium	1.11	2.72**
12	Skin & Subcutaneous Tissue	0.68*	0.54
13	Musculoskeletal System	1.11	0.54
14	Congenital Anomalies	1.70**	3.47**
15	Causes of Perinatal Morbidity & Mortality	1.73**	4.15**
16	Symptoms & Ill-defined	0.56*	0.47*
17	Accidents, Poisonings & Violence	0.78	0.29*
18	Supplementary Classifications	1.08	0.51

47 Diagnostic Categories

No	Diagnostic Category	Type 1	Type 2
1	Investigations, Procedures, Healthy	1.01	0.39
2	Infectious & Parasitic	0.64	0.55
3	Enteritis, Diarrhoeal Disease	0.41*	1.31
4	Malignant Neoplasms	1.62	1.01
5	Benign Neoplasms	1.22	0.59
6	Endocrine & Metabolic	0.81	0.25*
7	Blood	0.90	0.47
8	Psychiatric	1.31	1.22
9	Other CNS & Nerves	0.93	0.33*
10	Eye & Ear	1.24	1.06
11	Other Heart, Hypertension	0.91	0.99
12	Acute Myocardial Infarction	1.08	0.68
13	Symptomatic Heart Disease	0.82	0.67
14	Cerebrovascular Disease	1.12	0.67
15	Circulation	1.22	0.65
16	Upper Respiratory	0.40*	1.48
17	Pneumonia	0.49*	0.73
18	Bronchitis, Emphysema, Asthma	0.65	0.95
19	Tonsils & Adenoids	1.09	1.71
20	Other Respiratory	0.75	0.58
21	Dental	0.90	1.20
22	Upper Gastrointestinal	0.95	0.47
23	Appendicitis	0.84	0.57
24	Hernia	1.00	0.47
25	Other Gastrointestinal	1.00	0.43
26	Nephritis & Nephrosis	2.37**	2.82**
27	Other Urinary	0.93	0.38
28	Male Genital	1.10	0.45
29	Other Female Genital	1.01	0.67
30	Disorders of Menstruation	1.19	0.85
31	Complications, Pregnancy & Puerperium	0.89	1.77
32	Abortion	0.87	0.71
33	Normal Delivery	1.17	2.52
34	Delivery Complications	1.70**	4.75**
35	Skin Disease	0.64	0.41
36	Orthopaedic	1.04	0.41
37	Congenital Malformation	1.60	2.61
38	Perinatal	1.63**	3.08**
39	Immaturity	1.92**	4.47**
40	Symptoms, Ill-defined	0.53*	0.37
41	Long Stay, Ill-defined	0.62	0.85
42	Other Fractures (Excl. Femur)	0.91	0.30*
43	Fracture of Neck of Femur	1.46	1.21

APPENDIX (Continued)
STANDARDISED COMPLEXITY WEIGHTS, QUEENSLAND PUBLIC HOSPITALS, 1977-78
47 Diagnostic Categories

No	Diagnostic Category	Type 1	Type 2
44	Dislocations	0.67	0.58
45	Internal Injury	0.74	0.42
46	External Injury	0.61	0.41
47	Poisoning	0.71	0.27*

Notes: * = one of the four lowest complexity weights.
** = one of the four highest complexity weights.

Sources: Hospital Morbidity Data and information theory index.

NOTES AND REFERENCES

1. On the general problems associated with defining markets and industries see D. Needham, *The Economics of Industrial Structure Conduct and Performance,* Holt, Rinehart and Winston, London, 1978, Ch. 5 and references cited therein. For a more specific discussion of the problems of defining market structure in the hospital industry, see A. McGuire, 'The theory of the hospital: a review of the models', *Social Science and Medicine,* 20, 11, 1985, pp. 1177-84.
2. The current transfer of nurse education from hospitals to colleges of advanced education in Australia is an example of this. This is not to imply, of course, that the production of two or more of these broad output categories within a hospital is more costly than producing them in separate institutions. The point is simply that this is technologically feasible. The cost consequences of splitting off product lines in this way depend on whether there are economies or diseconomies of scope. See W.J. Baumol, J.C. Panzar and R.D. Willig, *Contestable Markets and the Theory of Industry Structure,* Harcourt Brace Jovanovich, New York, 1982 for a discussion of the concept of economies of scope.
3. For a detailed discussion of the meaning and measurement of hospital case mix, see M.C. Hornbrook, 'Hospital case mix: its definition, measurement and use: Part I. The conceptual framework', *Medical Care Review,* 39, 1, 1982, pp. 1-43 and M.C. Hornbrook, 'Hospital case mix: its definition, measurement and use: Part II. Review of alternative measures', *Medical Care Review,* 39, 2, 1982, pp. 73-123.
4. R.B. Fetter, Y. Shin, J.L. Freeman, R.F. Averill and J.D. Thompson, 'Case mix definition by diagnosis-related groups', *Medical Care,* 18, 2, 1980 (Supplement) is the definitive document on how DRGs are constructed.
5. A description of the DRG hospital payment scheme in the US can be found in B.C. Vladeck, 'Medicare hospital payment by diagnosis related groups', *Annals of Internal Medicine,* 100, 4, 1984, pp. 576-91.
6. G. Palmer, 'Hospital output and the use of diagnosis-related groups for purposes of economic and financial analysis', in J.R.G. Butler and D.P. Doessel (eds), *Economics and Health 1985: Proceedings of the Seventh Australian Conference of Health Economists,* School of Health Administration, University of New South Wales, Sydney, 1986, pp. 159-81; Health Department Victoria, *DRGs 1982-83 and 1983-84: Measurement of the output of Victorian public hospitals in 1982-83 and 1983-84 using diagnosis related groups,* Health Statistics Unit, Health Department Victoria, 1986;

Health Department Victoria, *DRGs 1984-85: Measurement of the output of Victorian public hospitals in 1984-85 using diagnosis related groups,* Health Statistics Unit, Health Department Victoria, 1986; P. Broadhead and S. Duckett, 'Death to the oxymoron: the introduction of 'rational hospital budgeting' in Victoria, or perhaps more accurately, an account of progress towards that goal', in J.R.G. Butler and D.P. Doessel (eds), *Economics and Health: 1987 Proceedings of the Ninth Australian Conference of Health Economists,* School of Health Administration, University of New South Wales, Sydney, 1988, pp. 22-33.

7. R.G. Evans and H.D. Walker, 'Information theory and the analysis of hospital cost structure', *Canadian Journal of Economics,* 5, 3, 1972, pp. 398-418.
8. Hornbrook, *op. cit.,* Part II, p. 104.
9. T.D. Klastorin and C.A. Watts, 'On the measurement of hospital case mix', *Medical Care,* 18, 6, 1980, p. 678.
10. H. Theil, *Economics and Information Theory,* North-Holland, Amsterdam, 1967.
11. The following simple numerical example illustrates these points. Suppose there are 10 hospitals in the system, i.e. N = 10. For the first index, the prior probability for each hospital is taken as (1/N) or 0.1 — there is a one-in-ten chance of any particular case being treated by any given hospital in the system. For the second index, the prior probability is taken as the proportion of all cases in the system treated by a given hospital, so that if one hospital of the ten treats 50 per cent of the cases then the prior probability for this hospital is 0.5. For each index the revised probability for any given hospital is the proportion of all cases in the system of a particular type, e.g. malignant neoplasms, treated by that hospital. Suppose a hospital treats 40 per cent of these cases. The revised probability is then 0.4. In the first index, the information gain for this hospital for this case type is $\ln(0.4/0.1) = \ln 4$, while in the second it would be $\ln(0.4/0.5) = \ln 0.8$ if this hospital treated 50 per cent of all cases in the system.
12. This is necessary because the unstandardised weights are sensitive to the size distribution of hospitals. A large hospital may tend to treat a large proportion of all cases in the system of a particular type simply because of its size but the unstandardised weight would read this as high concentration.
13. Evans and Walker, *op. cit.,* p. 401.
14. See Klastorin and Watts, *op. cit.,* p. 679 for an example of this.
15. M. Tatchell, 'Measuring hospital output: a review of the service mix and case mix approaches', *Social Science and Medicine,* 17, 13, 1983, pp. 871-83.
16. P.M. Tatchell, *An Economic Analysis of Hospital Costs in New Zealand,* unpublished PhD thesis, University of Waikato, New Zealand, 1977.
17. Tatchell, *op. cit.,* 1983, p. 876.
18. Klastorin and Watts, *op. cit.,* p. 679. See also Hornbrook, *op. cit.,* Part II, pp. 111-16.
19. Only one hospital was excluded because of data problems.
20. The relative stay index compares a hospital's actual with its expected average length of stay. The latter is calculated using the hospital's actual case mix and the state mean length of stay in each case mix category. For an explanation of this index see Ontario Hospital Services Commission, *Ontario Length of Stay Tables 1969-1971,* Ontario Hospital Services Commission, Toronto, Canada, January 1972; Ontario Hospital Services Commission, *The Relative Stay Index Report, 1971 Edition,* Ontario Hospital Services Commission, Toronto, Canada, February 1972; J.Leigh and A.J. McBride, 'Computing an index of relative hospital performance from an inpatient reporting system', in *Proceedings of the 6th Australian Computer Conference,* 1974, pp. 119-30; and Queensland Department of Health, *The Relative Stay Index: Queensland,* Planning and Development Unit, Division of Research and Planning, Queensland Department of Health, Brisbane, April 1980.
21. Standard deviation divided by the mean. The means are unweighted, i.e. they are calculated as the sum of the case mix proportions divided by the number of hospitals.
22. Normally, average cost per case equals the product of average cost per day and average length of stay but this is not so for the means in Table 3 because they are unweighted means.

23. These results are contained in the Appendix.
24. Evans and Walker, *op. cit.,* p. 402.
25. S.D. Horn and D.N. Schumacher, 'An analysis of case mix complexity using information theory and diagnostic related grouping', *Medical Care,* 17, 4, 1979, pp. 382-9.
26. P.R. Schapper, *An Economic Analysis of Hospital and Medical Services,* unpublished PhD thesis, University of Western Australia, Perth, 1984, Table 6.1, p. 132.
27. Evans and Walker, *op. cit.,* p. 408. Evans and Walker do not actually present results equivalent to those presented here, reporting only equations which include various scale and activity variables along with the information theory index. This quotation then pertains to equations which include the effects of these other factors.
28. Tatchell, *op. cit.,* 1977, pp. 295-6.
29. Horn and Schumacher, *op. cit.,* p. 386.
30. C.A. Watts and T.D. Klastorin, 'The impact of case-mix on hospital costs: a comparative analysis', *Inquiry,* 17, 4, 1980, pp. 357-67.
31. J. Hardwick, 'Hospital case mix standardisation: a comparison of the resource need index and information theory measures', in Butler and Doessel, *op. cit.,* 1986, pp. 36-63.
32. The relevant parameter estimates for the two Type 1 indexes are 618.87 and 644.61 using 18 and 47 diagnostic categories respectively.
33. Any differences in the results in these Tables, and the products of the weights in the Appendix and the relevant parameter estimate in the previous footnote, are due to rounding.
34. See, for example, A.W. Jenkins, 'Multi-product cost analysis: service and case-type cost equations for Ontario hospitals', *Applied Economics,* 12, 1, 1980, pp. 103-13 and J.R.G. Butler, *Hospital Cost Analysis,* unpublished PhD thesis, University of Queensland, Brisbane, 1988, Ch. 5.